University Chemistry Quiz 1

2014/10/02

1. (15%) Protons can be accelerated to speeds near that of light in particle accelerators. Estimate the wavelength (in nm) of such a proton moving at 2.90 x10^8 m s^{-1}.(The mass of a proton is 1.673 x10^{-27} kg.)

Strategy: We are given the mass and the speed of the proton and asked to calculate the wavelength. We need the de Broglie equation, which is Equation 1.20 of the text. Note that because the units of Planck's constant are J s, m must be in kg and u must be in m s^{-1} (1 J = 1 kg m^2 s^{-2}).

Solution: Using Equation 1.20 we write:

$$\lambda = \frac{h}{mu}$$

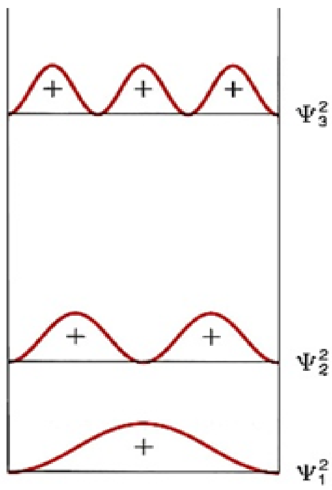
$$\lambda = \frac{h}{mu} = \frac{(6.63 \times 10^{-34}\ \text{kg m}^2\ \text{s}^{-1})}{(1.673 \times 10^{-27}\ \text{kg})(2.90 \times 10^{8}\ \text{m s}^{-1})} = 1.37 \times 10^{-15}\ \text{m}$$

The problem asks to express the wavelength in nanometers.

$$\lambda = (1.37 \times 10^{-15}\ \text{m}) \times \frac{1\ \text{nm}}{1 \times 10^{-9}\ \text{m}} = \mathbf{1.37 \times 10^{-6}\ nm}$$

2. (15%) Sketch the probability densities for the first three energy levels of the particle in a one-dimensional box. Without doing any calculations, determine the average value of the position of the particle (x) corresponding to each distribution.

Below are the probability distributions for the first three energy levels for a one-dimensional particle in a box, as shown in Figure 1.24. The average value for x for all of the states for a one-dimensional particle in a box is L/2 (the middle of the box). This is true even for those states (like the second state) that have no probability of being exactly in the middle of the box.



3. (15%) (a) What is the frequency of light having a wavelength of 456 nm?

(b) What is the wavelength (in nanometers) of radiation having a frequency of 2.45×10^9 Hz? (This is the type of radiation used in microwave ovens.)

(a).

Strategy: We are given the wavelength of an electromagnetic wave and asked to calculate the frequency. Rearranging Equation (1.2) of the text and replacing u with c (the speed of light) gives:

$$\nu = \frac{c}{\lambda}$$

Solution: Because the speed of light is given in meters per second, it is convenient to first convert wavelength to units of meters. Recall that $1 \text{ nm} = 1 \times 10^{-9}$ m. We write:

$$456 \text{ nm} \times \frac{1 \times 10^{-9} \text{ m}}{1 \text{ nm}} = 456 \times 10^{-9} \text{ m} = 4.56 \times 10^{-7} \text{ m}$$

Substituting in the wavelength and the speed of light ($3.00 \times 10^8 \text{ m s}^{-1}$), the frequency is:

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m}^{-1}}{4.56 \times 10^{-7} \text{ m}} = \mathbf{6.58 \times 10^{14}\ s^{-1}} \text{ or } \mathbf{6.58 \times 10^{14}\ Hz}$$

Check: The answer shows that 6.58×10^{14} waves pass a fixed point every second. This very high frequency is in accordance with the very high speed of light.

(b)

Strategy: We are given the frequency of an electromagnetic wave and asked to calculate the wavelength. Rearranging Equation (1.2) of the text and replacing u with c (the speed of light) gives:

$$\lambda = \frac{c}{\nu}$$

Solution: Substituting in the frequency and the speed of light ($3.00 \times 10^8 \text{ m s}^{-1}$) into the above equation, the wavelength is:

$$\lambda = \frac{c}{\nu} = \frac{3.00 \times 10^8 \text{ m s}^{-1}}{2.45 \times 10^9 \text{ s}^{-1}} = 0.122 \text{ m}$$

The problem asks for the wavelength in units of nanometers. Recall that $1 \text{ nm} = 1 \times 10^{-9}$ m.

$$\lambda = 0.122 \text{ m} \times \frac{1 \text{ nm}}{1 \times 10^{-9} \text{ m}} = \mathbf{1.22 \times 10^8\ nm}$$

4. (15%) A photon has a frequency of 6.0×10^4 Hz.

(a) Convert this frequency into wavelength (nm). Into what region of the electromagnetic spectrum dose this frequency fall?

$$\lambda = \frac{c}{\nu} = \frac{3.00 \times 10^{8}\ \mathrm{m}^{\ \ -1}}{6.0 \times 10^{4}\ \mathrm{s}^{-1}} = \mathbf{5.0 \times 10^{3}\ m} = \mathbf{5.0 \times 10^{12}\ nm}$$

(b) Calculate the energy (in joules) of this photon.

$$E = h\nu = (6.63 \times 10^{-34}\ \mathrm{J\ s})(6.0 \times 10^{4}\ \mathrm{s}^{-1}) = \mathbf{4.0 \times 10^{-29}\ J}$$

(c) Calculate the energy (in joules) of 1 mol of photons all with this frequency.

$$E = \frac{4.0 \times 10^{-29}\ \mathrm{J}}{1\ \mathrm{photon}} \times \frac{6.022 \times 10^{23}\ \mathrm{photons}}{1\ \mathrm{mol}} = \mathbf{2.4 \times 10^{-5}\ J\ mol^{-1}}$$

5. (15%) The retina of the human eye can detect light when the radiant energy incident on it exceeds a minimum value of 4.0 x10^{-17} J. How many photons dose this energy correspond to if the light has a wavelength of 600 nm?

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \mathrm{J\ s})(3.00 \times 10^{8}\ \mathrm{m\ s}^{-1})}{600 \times 10^{-9}\ \mathrm{m}} = 3.32 \times 10^{-19}\ \mathrm{J}$$

The number of photons needed to produce 4.0×10^{-17} J of energy is:

$$(4.0 \times 10^{-17}\ \mathrm{J}) \times \frac{1\ \mathrm{photon}}{3.32 \times 10^{-19}\ \mathrm{J}} = \mathbf{1.2 \times 10^{2}\ photons}$$

6. (15%) A photoelectric experiment was performed by separately shining a laser at 450 nm (blue light) and a laser at 560 nm (yellow light) on a clean metal surface and measuring the number and kinetic energy of the ejected electrons. Which light would generate more electrons? Which light would eject electrons with the greatest kinetic energy? Assume that the same number of photons is delivered to the metal surface by each laser and that the frequencies of the laser lights exceed the threshold frequency.

Because the same number of photons are being delivered by both lasers and because both lasers produce photons with enough energy to eject electrons from the metal, each laser will eject the same number of electrons. The electrons ejected by the blue laser will have higher kinetic energy because the photons from the blue laser have higher energy.

7. (15%) Consider the following energy levels of a hypothetical atom:

E_4: -1.0 x10^{-19} J

E_3: -5.0 x10^{-19} J

E_2: -10 x10-^{19} J

E_1: -15 x10^{-19} J

(a) What is the wavelength of the photon needed to excite an electron from E_1 to E_4?

$$E_4 - E_1 = (-1.0 \times 10^{-19}\ \text{J}) - (-15 \times 10^{-19}\ \text{J}) = 14 \times 10^{-19}\ \text{J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}\ \text{J s})(3.00 \times 10^{8}\ \text{m s}^{-1})}{14 \times 10^{-19}\ \text{J}} = \mathbf{1.4 \times 10^{-7}\ m} = \mathbf{140\ nm}$$

(b) What is the energy (in joules) a photon must have to excite an electron from E_2 to E_3?

$$E_3 - E_2 = (-5.0 \times 10^{-19}\ \text{J}) - (-10.0 \times 10^{-19}\ \text{J}) = \mathbf{5 \times 10^{-19}\ J}$$

(c) When an electron drops from the E_3 level to the E_1 level, the atom is said to undergo emission. Calculate the wavelength of the photon emitted in this process.

$$E_1 - E_3 = (-15 \times 10^{-19}\ \text{J}) - (-5.0 \times 10^{-19}\ \text{J}) = -10 \times 10^{-19}\ \text{J}$$

Ignoring the negative sign of ΔE, the wavelength is found as in part (a).

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}\ \text{J s})(3.00 \times 10^{8}\ \text{m s}^{-1})}{10 \times 10^{-19}\ \text{J}} = \mathbf{2.0 \times 10^{-7}\ m} = \mathbf{200\ nm}$$